Boundary-Layer Displacement and Leading-Edge Bluntness Effects on Attached and Separated Laminar Boundary Layers in a Compression Corner. Part II: Experimental Study

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In this study, heat transfer, pressure, skin friction, and schlieren measurements were made in attached and separated flows over flat plate-wedge compression surfaces in a hypersonic airflow. The effect on the compression corner flow of the favorable pressure gradient induced on a sharp flat plate upstream of the corner was studied for a range of freestream Mach numbers from 14 to 20 and unit Reynolds numbers from 2000/in. to 14,000/in., thus spanning the "weak" to "strong" interaction regimes ($1 < \bar{x}_L < 30$) over the flat plate. These studies have indicated that the length of laminar-separated regions increases with increasing Reynolds number and decreasing Mach number. Correlations of the wedge angle required to promote incipient separation and of the plateau pressure in well-separated flows are presented in terms of M_{∞} and x_{ϵ} . The effect of combined leading-edge bluntness and boundary-layer displacement on attached and separated regions was examined by the varying leading-edge configurations and freestream Reynolds number, to obtain conditions ranging from the displacement-dominated to the bluntness-dominated regime (10 > $x_{\epsilon}/\kappa_{\epsilon}^{2/3}$ > 0.08). blunting slightly increased the size of a separated region, whereas for $x_{\epsilon}/\kappa_{\epsilon}^{2/3} < 0.7$, the size of the well-separated region decreased with increased blunting. Correlations of the wedge angle required to promote incipient separation and of the plateau pressure in well-separated flow are presented in terms of $x_{\epsilon}/\kappa_{\epsilon}^{2/3}$.

Nomenclature

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= (\mu \partial u/\partial y)_{w}/\frac{1}{2}\rho_{\infty}u_{\infty}^{2}
                 q/\rho_{\infty}u_{\infty}(H_0-H_w)
                p/\rho_{\infty}u_{\infty}^2
                 constant of proportionality in \mu/\mu_{\infty} = CT/T_{\infty}
                 drag of the blunt leading edge
                symbol representing a functional relationship
                 D_N/\frac{1}{2}\rho_\infty u_\infty^2 t
                 reference length
                Mach number
p \\ Re
                 pressure
                 Reynolds number
                 thickness of blunt leading edge
                 T(1 + 3T_w/T_0)/6
                distance along the body surface
                C_p/C_v, specific heat ratio (\gamma - 1)/(\gamma + 1)
γ
θ
                 wedge angle
Kε
                M^{3}(C^{*}/Re_{L})^{1/2}, M^{3}(C^{*}/Re_{x})^{1/2}, respectively
\bar{x}_L, \bar{x}_x
                 \epsilon(0.664 + 1.73T_w/T_0)\bar{x}_L, \epsilon(0.664 + 1.73T_w/T_0)\bar{x}_x,
\chi_{\epsilon_L}, \chi_{\epsilon_x}
                    respectively
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Subscripts

L	= based on reference length	
t	= based on nose thickness	
W	= evaluate at the wall	
0	= based on freestream stagnation	n conditions
œ	= based on freestream condition	
*	= based on the reference temper	

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1. Introduction

LTHOUGH the classical problem of shock wave, boundary-ALTHOUGH the classical problem of and layer interaction has been studied extensively in subsonic and supersonic flows, only recently has the corresponding problem been examined at hypersonic speeds. The early studies of Bogdonoff and Vas, 1 and Sterrett and Emery2 showed the ease with which a laminar boundary layer separates and confirmed and extended some of the conclusions from the analyses and experimental work in supersonic flows by Chapman, Kuehn, and Larson³ on the occurrence and properties of laminar-separated regions. The effect of the Mach number and Reynolds number of the freestream on the occurrence, size, and properties of laminar-separated region over two-dimensional compression surfaces in hypersonic flow has been examined in a number of recent experimental studies.²⁻⁹ In most of these studies the pressure gradient induced by shock wave, boundary-layer interaction on the basic flat plate (the boundary-layer displacement effect) was small and no attempts were made to examine the effects of a pressure gradient upstream of the corner.

An important aspect of the study of boundary-layer development over compression surfaces is the effect of leading-edge bluntness. Only a very limited amount of experimental data is available on this topic. Both Townsend¹⁰ and Gray¹¹ conducted experimental studies to examine the effect of leading-edge bluntness on the size of separated regions on two-dimensional and axisymmetric models, respectively. But whereas Townsend found that increasing bluntness caused a decrease in the length of the separated region, Gray found the reverse trend. None of the previous studies explored the effect of combined boundary-layer displacement and leading-edge bluntness on the corner interaction from the displacement-dominated to the bluntness-dominated regimes.

One of the major problems that confronts those making an experimental study of bluntness effects is that many of the diagnostic techniques used to define the occurrence and size

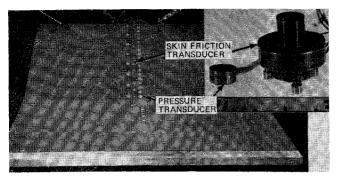


Fig. 1 2-ft span blunt flat plate-wedge model.

of separated regions are considerably less effective when examining the flow over strongly blunted bodies, than when used in the absence of an entropy layer. Schlieren techniques, which have proved invaluable in defining the outer edge of the viscous layer in the interaction region, are of little use in strongly blunted flows, where the entropy layer effectively masks out the density gradient associated with the edge of the boundary layer and the separation and reattachment process. The criteria for incipient separation and length of separated region developed from pressure and heattransfer measurements in the absence of bluntness are questionable because of the nonuniformity of entropy layer flow outside the viscous layer. Under these conditions, skinfriction measurements are one of the few measurements that can be made to determine the presence of separation and the precise position of the separation and reattachment points.

In the first part of the present experimental study, pressure, heat-transfer, and skin-friction measurements were made in attached and separated flows over sharp flat plate-wedge models. The test program was conducted over a range of freestream Mach numbers and unit Reynolds numbers so that they spanned the "weak" and "strong" interaction regions over the basic flat plate. Here, in addition to an examination and correlation of the properties of the interaction regions, particular emphasis was placed on a study of the time and mechanism to establish a steady separated flow, and the effects of the spanwise boundary conditions. In the second phase of the experimental study, designed to investigate leading-edge bluntness effects, skin-friction, heat-transfer, pressure, and schlieren measurements were made in attached and separated flow on the flat plate-wedge compression surfaces for a series of leading-edge configurations and freestream conditions. Here the experiments were designed so that the effect of combined bluntness and displacement on the corner interaction region could be evaluated over the complete range from the displacement-dominated to the bluntness-dominated regime. The measurements of the incipient separation condition, the plateau pressure, and the length of separated region were correlated in terms of nondimensional parameters determined from an examination of the similitude of these flows.

2. Experimental Program

The experimental investigation of shock wave, boundary-layer interaction on the flat plate-wedge models was conducted in the CAL 4- and 6-ft shock tunnels. $^{12.18}$ Both tunnels operated on tailored interface principles for the particular test conditions used in these studies, with useful running times of 5 and 4 msec, respectively. A photograph of one of the flat plate-wedge models used in the experimental studies are shown in Fig. 1. The flat plate consisted of a basic $12-\times12$ -in. instrumented section that could be fitted with a series of interchangeable leading edges, the width of which could be varied by the addition of side extensions to give model spans of 1, 1.5, and 2 ft. The flat plate was

hinged at its downstream end to a second flat instrumented section (the wedge or flap), 12 in. in length, the width of which was also adjustable. The wedge could be adjusted to any angle up to 40° relative to the flat plate. Both the flat plate and wedge were fully instrumented with heat-transfer, skin friction, and pressure transducers. The thin-film heattransfer gages, coated with a thin insulating film of titanium dioxide, were used in conjunction with analog networks. which converted their output directly into heat-transfer rate. Both orifice-type and flush-mounted pressure transducers¹⁴ were used in the model. The skin-friction transducers¹⁵ consisted of a 4-in. diam diaphragm mounted flush with the surface of the model, with the diaphragm connected by a flexure of a lead zirconium titanate crystal mounted in bending. This transducer, which is compensated for normal pressure, acceleration, and thermal effects, has a nominal sensitivity of 20 v/psi.

Three sets of tests to examine the attached and separated regions over the sharp flat plate-wedge models were conducted at nominal Mach numbers of 14, 16, and 20. In the first set of three tests, the 4-ft tunnel were run with a common stagnation temperature of 2900°K and reservoir pressures of 3900, 1400, and 500 psia, to give respective freestream conditions of $M=14.8,\ Re/\text{in.}=14,400;\ M=14.4,\ Re/\text{in.}=5500;\ and\ M=13.6,\ Re/\text{in.}=2200.$ The tests in the 6-ft tunnel at a Mach number of 16 were run at a stagnation temperature of 6000°K and a reservoir pressure of 3000 psia to yield a freestream Reynolds number of 2500/in., whereas the tests at Mach 19.8 used a stagnation temperature of 5000°K and a reservoir pressure of 12,000 psia to obtain a free-stream Reynolds number of 8700/in.

In the experimental program to investigate the effect of combined leading-edge bluntness and boundary-layer displacement, the 12-in. flat plate was fitted with five cylindrical leading edges having diameters of 1.200, 0.710, 0.200, 0.080, and 0.030 in. and two square leading edges having widths of 0.710 and 0.007 in. The program was conducted in the 6-ft tunnel at a Mach number of 19.8, a stagnation temperature of 5000°K, and reservoir pressures of 12,000 and 2400 psia to give freestream unit Reynolds numbers of 8700/in. and 2055/in.

2.1 Time Establishment of Attached and Separated Flows

An important part of the present experimental study was an investigation of the time and mechanism by which attached and separated flows in adverse pressure gradients are established. Very little is known about this subject. Although previous experiments¹⁶ had indicated that steady separated regions of flow were established within 10 m sec (the starting time of the gun tunnel in which the tests were conducted), and experiments and calculations by Rom¹⁷ had indicated as little as 100 µsec were required for a separated flow to stabilize, there have been no definitive experimental studies published on this problem.

The measurements made during this study were on a twodimensional model with a series of model widths, with and without side fences, to determine whether induced cross flow contributes to the time-dependent mechanism. thin-film heat-transfer gages and the skin-friction and flushmounted pressure gages had a response time of a small fraction of a millisecond. The experiments which were performed at Mach numbers from 14 to 20 and at Reynolds numbers from 2200/in. to 12,000/in. exhibited the same over-all pattern for the time and mode of flow establishment. The heat transfer, skin friction, and pressure measurements in attached flow over the model indicated that the boundary-layer flow had stabilized within the starting time of flow in the tunnel. Skin-friction, pressure, and heat-transfer records similar to those obtained under these conditions are shown in Fig. 2, position 1. It can be seen that steady levels are established

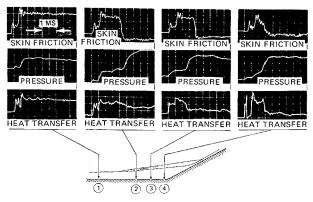


Fig. 2 Pressure and heat-transfer records during the establishment of a well-separated flow.

within 1 msec from the start of the flow in the tunnel, and the total test time is approximately 4 msec for these conditions.

When the strength of the interaction is sufficient to separate the flow, the records from gages in the separated region show that these flows take a considerably longer time to establish and are far more complex than attached flows in an adverse pressure gradient. As the leading edge of the interaction moves upstream, it crosses the gages at station 3, causing the pressure to rise and the heat transfer and skin friction to drop from their flat-plate values to steady levels corresponding to the well-separated conditions. negative skin-friction value clearly identifies the reversed flow adjacent to the wall in the fully established, wellseparated flow. An examination of these records, and many like them, indicates that a small separated region is created during the starting process of the tunnel; this region grows in size during the steady running time to reach an equilibrium approximately 2 msec after the start of steady airflow. Our measurements for a variety of freestream conditions, for wedge angles which promote well-separated flows, and for the complete range of spanwise configurations lead to the conclusion that the time and mechanism for establishing a separated flow over these models is associated with establishing a recirculating flow in a streamwise plane perpendicular to the surface of the model, rather than a phenomenon associated with establishing a lateral outflow or recirculation in a plane parallel to the surface of the model. Observations from many tests indicate that the leading edge of the separated region propagates forward at approximately the local speed of sound close to the wall.

2.2 Finite Span Effects

The general approach adopted here has been to vary the transverse boundary conditions, by the addition of spanwise extensions and side fences, until a successive change produced no measurable effect on the longitudinal or transverse distribution of heat transfer, skin friction, or pressure on the model. When these conditions are fulfilled, then it is believed that the flow over the center of the model can be described as two-dimensional. The studies of Walker¹⁸ and Lewis, Kubota, and Lees⁹ strongly support this viewpoint. Measurements were made of the longitudinal and spanwise distribution of heat transfer and pressure for model widths of 1, 1.5, and 2 ft. Side fences were also added to investigate the effect of preventing outflow from the sides of the model. The side fences, which were triangular in shape, were mounted to the sides of the models with their upper edge along the top of the shear layer.

From present tests we have found the following trends. For flows where the adverse pressure created by the wedge was of insufficient strength to separate the flow (i.e., the

wedge angle was less than the one required to induce incipient separation), increasing the model span and adding side fences did not significantly influence either the longitudinal or transverse distribution of properties of the interaction regions over the models. Adding side fences to the 1-ft span model increased the size of the largest separated region beyond its size on the 1.5- and 2-ft span models without side fences; however, the addition of side fences to the models with spans of 1.5 and 2 ft had no measurable effect on the size and properties of the separated regions.8 All the experimental data presented herein were obtained on models with a span of 2 ft. It was observed that in the tests where increasing the span of the model increased the size of the separated region, there were no significant changes found in the form of the transverse distribution of heat transfer and pressure, although a measurable change occurred in the absolute level. The tests have demonstrated the great importance of investigating finite span effects by varying the spanwise configuration of the models. Great care must be exercised in the design and use of side fences, for the addition of side fences can cause as much flow distortion as the lack of them.

3. Shock Wave Boundary-Layer Interaction on Sharp Flat Plate-Wedge Models

In most of the earlier experimental studies, the pressure gradients generated on the basic flat plate resulting from boundary-layer displacement effects have been small. In the present study, we have examined the properties of both attached and separated regions at a series of freestream conditions that produced conditions on the flat plate at its junction with the wedge, which span the range from the weak to the strong interaction regime. We examined the effect of the Mach number and unit Reynolds number of the freestream and the viscous interaction parameter χ_{ϵ} on the occurrence, size, and properties of the corner interaction regions.

A correlation of the heat-transfer and pressure measurements for the flow over the basic flat plate is shown in Fig. 3. Here we see that the measurements were obtained in the range from $0.8 < \chi_{\epsilon} < 4.0$, which spans viscous weak and strong interaction regimes. Both the pressure and the heat-transfer measurements obtained in the present study fall slightly above the theory in both regimes, a feature which has been observed in a number of earlier experimental studies. This discrepancy is due in part to the fact that the theory is based on local flat plate similarity which uses the Blasius profiles to describe the velocity and temperature in the boundary layer.

Because it is not possible to publish here all the measurements obtained in the present study, we have chosen a number of distributions which illustrate the general features of attached and separated flows and the way in which they develop. A more complete record of the measurements obtained in this work is published in Ref. 8.

3.1 Presentation and Discussion of the Experimental Measurements

Some typical distributions of skin friction, pressure and heat transfer in attached, incipient separated and well-separated flows are shown, together with schlieren photographs, in Fig. 4. Throughout the paper, only the important centerline measurements and schlieren photographs from the tests with the 2-ft span model without side fences are shown. Figure 4a shows the distribution for attached flow over the model. The wedge does not influence the skin-friction, pressure or heat-transfer distributions upstream of the hinge line. Each distribution displays a uniform increase to reach a maximum of 8 in. downstream of the hinge, and then a subsequent decay to the end of the plate. The distribution for conditions close to the incipient separation are shown

in Fig. 4b. Here the skin friction is zero first on the wedge close to the hinge line. In these high Mach number, low Revnolds number flows over the highly cooled compression surfaces, most of the momentum is concentrated at the outer edge of the boundary layer. Thus the thick attached boundary layer does not experience a large pressure gradient until downstream of the hinge line where the outer edge of the viscous layer begins to turn parallel to the wedge. interaction has just begun to influence the properties on the flat plate, causing the heat transfer and skin friction to drop sharply below their flat-plate values just upstream of the hinge to form a cusp-like distribution in this region. The pressure increases from the flat-plate value just upstream of the hinge line, with a point of inflexion close to the separation point. The pressure and heat transfer and skin friction reach a maximum at the same point before each decays towards the trailing edge of the wedge. A typical distribution of properties in well-separated flow is shown in Fig. 4c. In this flow, the interaction has fed well upstream of the hinge, causing the pressure, skin friction and heat transfer to depart markedly from their flat-plate values. The pressure rises abruptly at the beginning of the interaction on the flat plate to reach an approximately constant level (the plateau pressure) just upstream of the hinge line. In this region, the skin friction drops sharply to zero at the separation point, and reaches an approximately constant negative level in the plateau region just ahead of the wedge. Separation occurs at 0.6 of the plateau pressure rise slightly downstream of the inflexion point. The skin friction becomes increasingly negative along the wedge until it rises to zero at reattachment, and then to a maximum at the end of the reattachment pressure rise in an extremely short distance. Reattachment occurs at approximately 0.6 of the reattachment pressure rise for this case.

Figures 5a and 5b show the development of pressure and heat-transfer distributions in the Mach 16, $\tilde{\chi}_L = 19.8$ air-flow as the wedge angle is varied. As in the studies at Mach

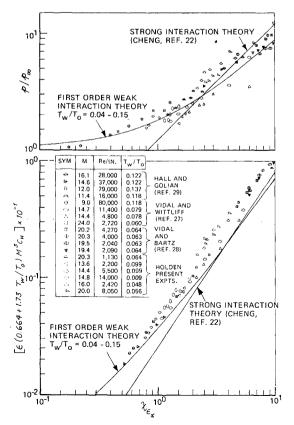


Fig. 3 Correlation of pressure and heat-transfer data on sharp flat plates.

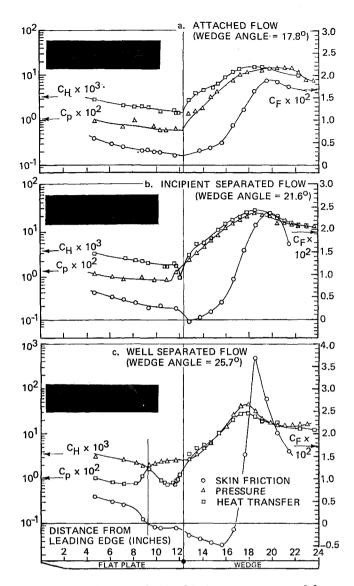


Fig. 4 Distribution of skin friction, pressure, and heat transfer on the sharp flat plate-wedge model ($M_{\odot}=19.8$, $Re/\text{in.}=8.70\times10^3$).

14 and 20, the characteristic change in the pressure distribution occurs with increase in the strength of interaction, from a uniform rise along the wedge in an attached flow to the "stepped" pressure distribution over the flat plate-wedge, typical of laminar well-separated flow. The existence of a well-defined plateau pressure region in all the measurements made in well-separated flow cends to clarify the picture presented by Gray, 19 who questioned the existence of a plateau region in fully laminar separated flow. The measurements made in this study indicated that the plateau pressure increased with increasing Mach number and decreasing Reynolds number. A correlation of the plateau pressure measurements in terms of the viscous interaction parameter χ_{ϵ} . (derived from viscous hypersonic similitude; see section 4.1) is shown in Fig. 6. The heat-transfer measurements made in the present study are similar in form to those found in earlier experimental work. 4.5 The heat-transfer distribution in attached flows is characterized by a cusp-like distribution in the interaction region close to the flat plate-wedge junction, while the heat-transfer distribution in well-separated flow has a rounded concave form in this region.

The characteristics of the interaction regions that we have studied in the present work differ in a number of ways from those exhibited in the supersonic high Reynolds number regime. The most notable feature is the very small upstream influence of the adverse pressure gradient created by

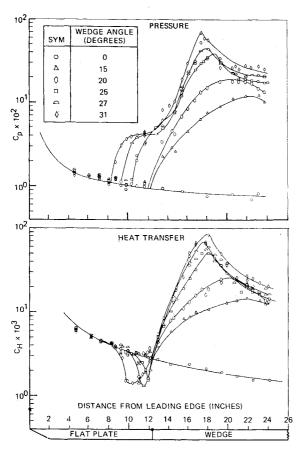


Fig. 5 Variation of the distribution of pressure and heat transfer on the flat plate-wedge model as the strength of the interaction is varied ($M_{\infty} = 16$; $\bar{x}_L = 19.8$).

the wedge on the attached flow over the flat plate. The extent of this upstream influence was found to decrease with increasing Mach number and decreasing Reynolds number. Increasing the wedge angle beyond that required to promote incipient separation causes the interaction to feed forward, markedly influencing the distribution of properties on the flat plate. The abrupt change in the size and properties of the interaction region, as the viscous layer changes from attached to separated, is in contrast with measurements in the supersonic and low hypersonic range where, although the same characteristic changes occur in the distribution of pressure and heat transfer, the size and development of the interaction region increase less rapidly and more uniformly. The separated flows in the present study exhibit the same major features which characterize laminar separated flows at lower Mach number, namely, strong localized interactions in the separation and reattachment regions bounding a region of constant pressure (the plateau pressure region). However, gradients of properties in the separation and reattachment regions are of greater severity, with the compression fans generated in these regions coalescing almost immediately to form strong shocks. The measurements at constant Mach number show that decreasing freestream Reynolds number decreases the size of the separated region, although the trend is weak. Measurements at a constant freestream Reynolds number indicate that increasing Mach number causes a decrease in the size of the separated region. Increasing Mach number and decreasing Reynolds number also had an inhibiting effect on flow separation.

3.2 Incipient Separation

One of the objectives of this work was to determine the conditions required to induce the incipient separation of a laminar boundary layer in the high Mach number, low Reynolds number flow regime. The incipient separation

condition is defined when there is one point only in the interaction region where the skin friction is zero. Earlier experimental and theoretical studies2,4,7,21 defined the incipient separation condition on the basis of the first appearance of an inflexion in the pressure distribution or a change in the heat-transfer distribution from a cusp-like shape to a rounded concave distribution in the interaction regions. The measurements of skin friction, heat transfer and pressure, together with schlieren photographs obtained in the present study, support the conclusions of this previous work. On the basis of the measurements made here, it is estimated that the wedge angle required to promote incipient separation (θ_{incip}) was determined to within $\pm 5\%$. Based on a suggestion by Hall,24 some earlier measurements of incipient separation^{7,20} were correlated in terms of $M_{\infty}\theta_{\rm incip}$ and $\overline{\chi}_L$, the viscous interaction parameter based on the length of the flat plate. From the relationships developed in Sec. 4.1, we find that the important parameters governing the hypersonic similitude of the flow of the incipient separation condition are $(M_{\infty}\theta_{\rm incip})^2/\chi_{\rm cL}$. Figure 6 shows that the measurements from this and earlier studies correlate well when plotted in this form. The correlation has been verified directly in the present study by performing measurement at Mach numbers of 16 and 20 with the same value $\chi_{\epsilon L}$ (= 2.5). The measurements compare well with calculations made using the present theoretical analysis (Part 1) and the theoretical method proposed by Hankey.²³ Both theory (viscous hypersonic similitude and solutions to the integral forms of the boundarylayer equations) and experiment indicate that $M_{\infty}\theta_{\rm incip}$ \propto $\chi_{\epsilon_L}^{1/2}$ or $\theta_{\rm incip} \propto (M_{\odot}/Re_L^{1/2})^{1/2}$.

4. Shock Wave Boundary-Layer Interaction on Blunt Flat Plate-Wedge Models

4.1 Introduction

In practical designs for compression surfaces on hypersonic vehicles, some degree of leading-edge bluntness must be incorporated to control the heating in this region. well as influencing the local inviscid flowfield over the compression surface, leading-edge bluntness may have a strong effect on the viscous-inviscid interaction and the occurrence and extent of flow separation. The major effect of leadingedge bluntness is to create a region of high-temperature, lowdensity gas that has been processed by the strongly curved shock at the nose. This layer of gas (the entropy layer) expands downstream, creating a region of high-temperature, rotational inviscid flow between the viscous layer and the strong leading-edge shock. Thus, in these flows the viscous layer grows by mutual interaction with the entropy layer. The development of the viscous layer on the flat plate and in the corner interaction region is governed by the combined effects of boundary-layer displacement and leading-edge bluntness, and our problem here is to determine and explore

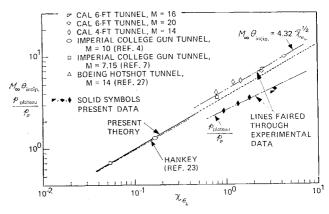


Fig. 6 Correlation of incipient separation and plateau pressure on a sharp flat plate-wedge model.

the effects of the important parameters which characterize these flows. The problem of describing the development of a boundary layer over a blunt flat plate has been considered by many investigators. Perhaps one of the most successful and elegant analytical models of these flows was given by Cheng.²² In addition to obtaining solutions for combined leading-edge bluntness and boundary-layer displacement effects, Cheng discussed the general viscous hypersonic similitude including the effects of leading-edge bluntness. He showed that the general similitude governing the boundary-layer thickness, surface pressure, skin friction and heat transfer around similar slender two-dimensional bodies can be written in terms of the nondimensional parameters

$$\frac{C_F}{\theta^3}\,\frac{C_H}{\theta^3}\,\frac{p_w}{p_\infty}\,(M^2\theta^2),\,=\,f\!\left(\!\frac{x}{L},\,\frac{x_{\epsilon L}}{M^2\theta^2},\frac{1}{2}\,M\theta\,\frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}^2},\,\frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}^2}\gamma,\,\frac{\mathrm{T_w}}{\mathrm{T_o}},\,\mathrm{Pr}\right)$$

or, in the alternative form,

$$\begin{split} M^3 C_F \!\!\left(\!\frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}^2}\!\right)^{\!\!3}, \, M^3 C_H \!\!\left(\!\frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}^2}\!\right)^{\!\!3}, \, & \frac{p_w}{p_\omega} \!\!\left(\!\frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}^2}\!\right)^{\!\!2} = \\ f\!\!\left[\!\frac{x}{L}, \, \frac{x_{\epsilon_L}}{\kappa_{\epsilon_L}^{2/3}}, \, \frac{1}{2} \!\!\left(\!\frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}}\!\right) \!\! M\theta, \, \frac{\kappa_{\epsilon_L}}{\chi_{\epsilon_L}}, \, \gamma, \, \frac{T_w}{T_o}, \, \Pr\right] \end{split}$$

Without making experiments or detailed flowfield calculations, it is possible to examine some of the effects that leading-edge bluntness will have on the corner interaction region and the occurrence of separation. Introducing leading-edge bluntness increases the magnitude of the favorable pressure gradient upstream of the corner and, consequently, causes velocity distribution on the boundary layer to be "fuller" close to the body. The Mach number at the edge of the boundary layer is greatly reduced by even a moderate amount of bluntness, and although the pressure is increased by blunting, the large increase in temperature of the gas as it passes through the nearly normal shock drives the density and the local unit Reynolds number down. While the favorable pressure gradient and lower local unit Reynolds

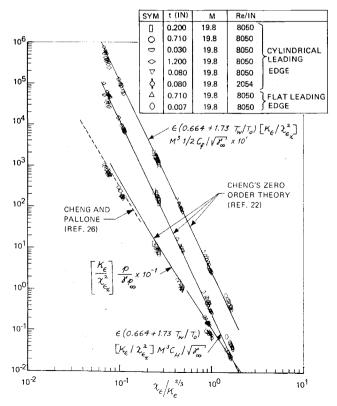


Fig. 7 Correlation of skin friction, heat transfer, and pressure data on blunted flat plates.

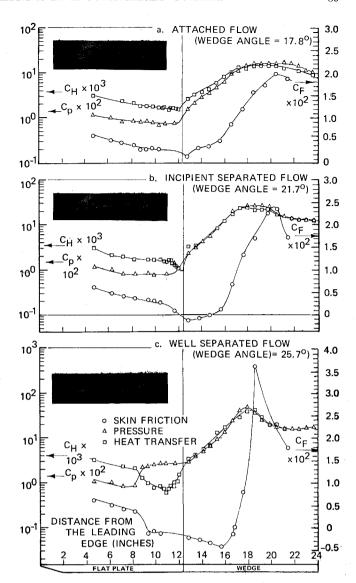


Fig. 8 Distribution of skin friction, pressure, and heat transfer on the 0.007-in. flat-ended blunt flat plate-wedge model ($M_{\infty}=19.8,\,Re/in.=8.70\times10^3$).

number tend to stabilize the attached layer, the locally lower Mach number is a strong destabilizing influence; thus, the effect of bluntness on the occurrence of incipient separation is difficult to predict a priori. However, features which should be evident in moderate-to-strongly-blunted flow are a significant reduction in pressure recovered on the wedge and a masking of many of the features of the reattachment compression process.

4.2 Presentation and Discussion of the Experimental Measurements

In this experimental study, the thickness of the leading edge was varied to obtain conditions on the flat plate ahead of the wedge from the displacement-dominated regime to the bluntness-dominated regime. Both flat and hemicylindrical leading edges were used to examine whether the structure of the entropy layer had a measurable effect on the occurrence and properties of separated flows. The effect of the unit Reynolds number of the freestream on the properties of the interaction regions was also examined. A comparison between Cheng's zeroeth order theory and the experimental measurements on the basic flat plates is shown in Fig. 7. Here we see there is good agreement between theory and experiment over the complete range of the combined blunt-

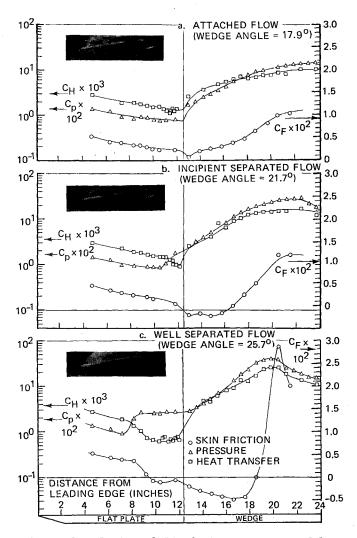


Fig. 9 Distribution of skin frction, pressure, and heat transfer on the 0.080-in. diam blunt flat plate-wedge model $(M_{\infty} = 19.8, Re/in. = 8.70 \times 10^3)$.

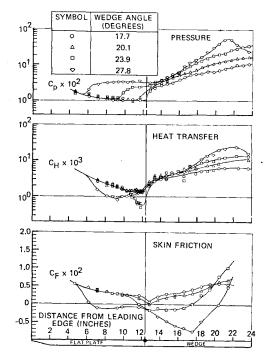


Fig. 10 Distribution of skin friction, pressure, and heat transfer on the 0.20-in. diam blunt flat plate-wedge model $(M_{\infty} = 19.8, Re/in. = 8.70 \times 10^3)$.

ness-viscous interaction parameter $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3}$ from the bluntness-dominated regime, $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} < 0.1$, where the pressure measurements are in agreement with the theory of Cheng and Pallone, 26 to the displacement-dominated regime $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} > 1$ where the strong interaction theory is valid. The measurements for different nose shapes correlate well, however, at very low Reynolds numbers in these highly cooled flows. Cheng's theory 22 appears to overestimate the contribution of boundary-layer displacements and, as a result, the theory overestimates the pressure on the flat plate.

The measurements obtained for the series of selected bluntness conditions are shown in Figs. 8-11. The first signs of the influence of leading-edge bluntness on the character of the flow in the corner occur for a leading-edge thickness of 0.007 in. We find that the size of the separated region, when the flow is close to the incipient separation condition, is increased slightly by blunting. The structures of the attached and well-separated flows are not significantly affected by this degree of bluntness. When the leadingedge bluntness is increased to 0.080-in. diam, the degradation of the total pressure in the entropy layer is reflected in the slower pressure rise along the wedge in the attached flow (Fig. 9a) and in a general decrease in the level of the skin friction and heat transfer on the wedge; these distributions no longer exhibit a local maximum on the wedge. Again, the separated flow at the condition closest to incipient separation is influenced the most by leading-edge bluntness, with the separated region over the wedge growing to almost 4 in. in length. The addition of moderate blunting to the wellseparated flow promoted by the 26° wedge (Fig. 9c) causes the reattachment point to move downstream along the wedge, thus causing an increase in the size of the separated

For the 0.20-in. diam blunt flat plate-wedge, it can be seen that a 20.17° wedge angle promotes incipient separation. Increasing the wedge angle causes the separated region to increase in size, first by the movement of the reattachment point downstream, creating a separated region which is situated almost entirely over the wedge; then, a further increase in wedge angle causes the separation point to move rapidly forward, creating a large separated region symmetrically placed about the hinge line. A marked reduction of the skin friction in the attached flow over the wedge results from blunting the leading edge. The pressure measurements show clearly that in these flows the compression of the non-

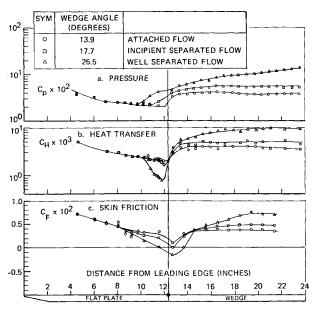


Fig. 11 Distribution of skin friction, pressure, and heat transfer on the 0.71-in. flat-ended blunt flat plate-wedge model conditions ($M_{\infty}=19.8$, $Re/in.=8.7\times10^3$).

uniform flow in the entropy layer is as important a factor in determining the structure of the flow after the wedge as the viscous-inviscid interaction in this region. In all but the largest region of separated flow, the pressure increases in a linear manner from the beginning to the end of the wedge, giving little evidence of the existence of the reattachment compression process. However, well-separated flow over the flat plate is still characterized by a plateau pressure region, a region where the heat transfer is reduced to approximately half the local flat-plate value.

Figure 11 shows the distributions over the 0.71-in. flatended blunt flat plate-wedge compression surface, a configuration for which the flow over the flat plate is dominated by leading-edge bluntness. The skin-friction measurements indicate that the wedge angle to promote incipient separation has been reduced to approximately 17.8°, and increasing the wedge angle by 8° promotes a separated region which is only 2 in. in length. The loss in the dynamic pressure through the bow shock has been so great that the skin friction, heat transfer, and pressure on the wedges are of the same magnitude as the measurements on the flat plate. For these wedge angles, the skin friction, heat transfer, and pressure are approximately constant over the entire length of the wedge. The pressure distribution in the well-separated flow no longer has a well-defined plateau region, and the increase in the pressure along the wedge is believed to be related to the compression of the nonuniform stream rather than to the structure of the reattachment compression process.

In the present study, the sizes of the separated regions were determined from the skin-friction measurements; it is clear that in these blunt plate experiments it would be difficult to infer the length of the separated region from either pressure or heat-transfer distributions. Figure 12 shows the variation of the length of a well-separated region (normalized by the distance to the start of separation) with displacement-bluntness interaction parameter $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3}$ over the flat plate-25.67° wedge model. We find the effect of a slight-to-moderate degree of blunting $(\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} > 0.5)$ is to increase slightly the size of the interaction region, whereas for $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} < 0.5$, increasing the leading-edge bluntness causes a rapid decrease in the size of the separated region. The effect of leadingedge bluntness is to increase the favorable pressure gradient over the flat plate, at the same time causing a reduction in the local Mach number and Reynolds number. In Sec. 3.1 we found that lowering the Mach number and increasing the unit Reynolds number increased the size of a separated region and decreased the wedge angle required to promote incipient separation, whereas increasing the favorable pressure gradient had the opposite effect. The present experiments indicate that for small bluntnesses, the reduction in the local Mach number outweighs the effect of a decrease in the Reynolds number and, thus, an increase in the size of the separated region results, whereas at the larger bluntnesses, where the changes in local Mach number and Reynolds number for a change in bluntness are small, the effect of the strong favorable pressure gradient is predominant. Since the wedge angle to promote incipient separation decreases slightly with increased bluntness, very small separated regions close to the incipient separation condition behave in a more complicated manner in highly blunted flow. The major effect of strong bluntness is to cause a decrease in the scale of the viscous-inviscid interaction in the corner.

The results of the present study are in agreement with Townsend, who obtained measurements on blunted plates for $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} < 0.1$, and found that increasing the bluntness of the flat plate decreased the size of the separated region. He explained this result by reference to the work of Gray, which indicated that lowering the Mach number and increasing the Reynolds number decreased the size of the separated region; Townsend suggested the Mach number effect was dominant and, hence, the size of the separated region decreased with increasing bluntness. Unfortunately, in fully laminar flow,

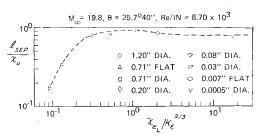


Fig. 12 Variation of the length of well-separated regions with the bluntness viscous interaction parameter $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3}$.

the variations in the length of a separated region with Mach number and Reynolds number are opposite to those found by Gray (Gray's conclusions were based on transitional data). A study of the effect of leading-edge bluntness on the separated flow over a flat plate-wedge in helium at Mach 11 by Graham and Vas² indicated the separated regions were very sensitive to small changes in leading-edge thickness (for $t=0.5\times10^{-3}$), which contrast with the present results for small bluntness. They found a decrease in the size of the interaction region with increased bluntness, but cautioned that the flows may not have been completely laminar.

In a more recent experimental study, Gray¹¹ found that slightly blunting a flared cone configuration resulted in an increase in the size of the separated region, which is the same trend observed in the present study for slightly blunted flat plates. However, a direct comparison between these results and those from the two-dimensional studies must be made with care, for it is well known that a blunted cone has a pressure undershoot near the nose, downstream of which an adverse pressure gradient exists for a finite distance. Thus, in some cases, cone bluntness can cause an adverse pressure gradient at the coneflare junction in addition to reducing the local Mach number and Reynolds number. In Gray's work, the cone of smallest bluntness had an adverse pressure gradient on its surface close to the junction with the flare (which itself enhances separation); for the largest bluntness the pressure gradient, although favorable, was comparable with the weak favorable pressure gradient on the flat plates in the present studies for $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} > 0.5$. Thus, it appears an increase in the size of the separated regions in Gray's study is consistent with the measurements and explanation reported here.

We found, as did Townsend, that an increase in leading-edge bluntness decreased the plateau pressure ratio p_{plateau}/p_0 . The reduction in the plateau pressure is consistent with the dominant effect of the reduction in local Mach number resulting from leading-edge bluntness. The measurements of plateau pressure have been correlated in Fig. 13 in terms of the viscous hypersonic similitude parameters (p_{plateau}/p_0) $(\kappa_{\epsilon_0}/\chi_{\epsilon_0}^2)^2$ and $\chi_{\epsilon_0}/\kappa_{\epsilon_0}^{2/3}$, which can be calculated directly from the model configuration, distance to the beginning of the interaction (x_0) , and freestream condition. It is interesting to note that the plateau pressure measurements on the models with the 1.2-in. diam and the 0.71-in. flat leading edge (which have the same nose drag) were almost identical, indicating that the shape of the leading edge is not an important parameter in these flows.

4.3 Incipient Separation

The effect of leading-edge bluntness is to decrease the local Mach number, which would bring the boundary layer closer to separation, while at the same time lowering the local Reynolds number and increasing the favorable pressure gradient, both of which exert a stabilizing influence on an attached boundary layer. Thus, the variation of the wedge angle to induce incipient separation with leading-edge bluntness was not expected to be large.

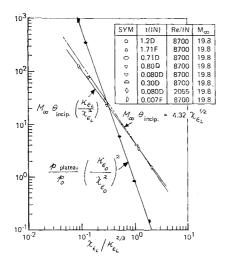


Fig. 13 Correlation of the incipient separation and plateau pressure on blunt flat plate-wedge compression surfaces.

We used skin-friction measurements to determine the incipient separation condition; however, we also attempted to correlate incipient separation with the heat-transfer and pressure measurements as we have done in earlier studies. However, detecting incipient separation on the basis of observing the first occurrence of a point of inflexion in the pressure distribution was found to be inaccurate for moderately blunted flows and practically impossible in flows strongly influenced by leading-edge bluntness.

In order to correlate the conditions required to induce incipient separation, we appeal to relationships governing the hypersonic similitude proposed by Cheng and given in sect. 4.1. For the purpose of developing the similitude for incipient separation, we assume (on the strength of experimental observations) that the incipient separation condition can be defined as the condition where $C_F = 0$ when x = L and $\theta = \theta_{\text{incip}}$. Thus, for a fixed T_w/T_o the similitude relating the parameters governing incipient separation is of the form:

$$f_{1}\left[\frac{\chi_{\epsilon_{L}}}{(M\theta)_{\text{incip}}^{2}}, \frac{M\theta_{\text{incip}}}{2} \frac{\kappa_{\epsilon_{L}}}{\chi_{\epsilon_{L}^{2}}}, \frac{\kappa_{\epsilon_{L}}}{\chi_{\epsilon_{L}^{2}}}\right] = 0 \text{ and}$$

$$f_{2}\left[\frac{\chi_{\epsilon_{L}}}{\kappa_{\epsilon_{r}^{2}/3}}, \frac{M\theta_{\text{incip}}}{2} \left(\frac{\kappa_{\epsilon_{L}}}{\chi_{\epsilon_{L}}}\right), \frac{\kappa_{\epsilon_{L}}}{\chi_{\epsilon_{L}}}\right] = 0$$

For the case of a sharp flat plate-wedge, $\kappa_{\epsilon}=0$ and the similitude reduces to the form

$$f_1[\chi_{\epsilon_I}/(M\theta)_{\rm incip}^2] = 0$$

Thus, we find that $M\theta_{\text{incip}} \propto \chi_{\epsilon_L}^{1/2}$. For the blunted flat plate-wedge, the parameters governing the incipient separation conditions are

$$M\theta_{\rm incip}(\kappa_{\epsilon_L}/\chi_{\epsilon_L}^2)$$
, $\chi_{\epsilon_L}/\kappa_{\epsilon_L}^{2/3}$, and $\kappa_{\epsilon_L}/\chi_{\epsilon_L}^2$

In Fig. 13, we have correlated all the measurements of incipient separation made in the present study in terms of $M\theta_{\rm incip}(\kappa_{\epsilon_L}/\chi_{\epsilon_L}^2)$ and the bluntness-viscous interaction parameter $\chi_{\epsilon_L}/\kappa_{\epsilon_L}^{2/3}$. We find that over the entire range, increasing the leading-edge bluntness (or constant freestream Mach number and unit Reynolds number) causes a decrease in the wedge angle to induce incipient separation. The relatively weak effect that bluntness has on $\theta_{\rm incip}$ can be seen by comparison with the curve $M_\infty\theta_{\rm incip}=4.32~\chi_{\epsilon_L}^{1/2}$ (plotted in Fig. 13), which would be the variation if leading-edge bluntness effects were negligible. Thus, it would

appear that the destabilizing effect of the locally lower Mach number at the corner on an attached boundary layer outweighs the stabilizing influence of a decrease in the Reynolds number and of an increase in favorable pressure gradient. It is interesting to note that if, as was indicated by the study of incipient separation on sharp flat plates, $(M/Re_L^{1/2})^{1/2}$ is the important parameter governing $\theta_{\rm incip}$, then the approximate calculation of this quantity on blunted flat plates, based on assuming an isentropic expansion from the stagnation point, yields the result that $\theta_{\rm incip} \propto 1/(kt)^{(\gamma+1)/12\gamma}$. The correlation shown in Fig. 13 indicates that $M\theta_{\rm incip} = 4.34\kappa_\epsilon^{-1/15}\chi_{\epsilon L}^{3/5}$.

5. Concluding Remarks

In this paper an experimental study has been described of leading-edge bluntness and boundary-layer displacement effects on laminar flow in a compression corner in high-temperature, hypersonic air flow. In this study we made skin-friction, heat-transfer, pressure and schlieren measurements in attached and separated regions for a series of model geometries, at unit Reynolds number from 2000 to 14,000/in. and Mach number from 14 to 20. It was found in this shock tunnel study that separated regions could take as long as 2.0 msec to establish. The effects of model span were investigated for a series of model widths with and without side fences. It was found that neither extending to span beyond two feet or adding side fences had an effect on the centerline measurements.

The measurements in the corner region of the sharp flat plate-wedge models revealed that the upstream influence of the wedge in attached flow was extremely small. However, when the flow separated, the interaction fed forward very rapidly with increasing wedge angle, in marked contrast with the characteristics of laminar boundary layers in supersonic flow, where the upstream influence increased uniformly with wedge angle for both attached and separated flow. For a fixed wedge angle the length of the separated region decreased with increasing freestream Mach number and decreasing unit Reynolds number. A region of constant pressure was found in all well-separated flows and the plateau pressure was correlated in terms of χ_{ϵ_0} . The incipient separation was determined at each freestream condition from the skin-friction, heat-transfer, and pressure distribu-The skin-friction measurements verified the separation criteria, based on characteristic changes in the form of the heat-transfer and pressure distribution, which had been proposed in earlier studies. It was found that those measurements correlated in the form $M_{\infty}\theta_{\rm incip} \propto 4.32 \chi_{\epsilon_L}^{1/2}$.

The study of the interaction regions on blunted flat platewedge was performed at a series of model configurations and freestream conditions to obtain conditions on the flat plate from the viscous- to the bluntness-dominated regime (0.08 < $\chi_{\epsilon_L}/\kappa_1$, $^{2/8}$ < 2). The measurements of skin friction, heat transfer, and pressure on the basic flat plate agreed well with Cheng's theory. For well-separated flows, increasing the leading-edge bluntness for $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} > 0.5$ slightly increased the size of the separated flow, whereas for $\chi_{\epsilon}/\kappa_{\epsilon}^{2/3} < 0.5$, an increase in bluntness caused a marked decrease in the length of separated region. The plateau pressure ratio decreased with increased bluntness and correlated in the form $(p_{plateau}/$ p_0) $(\kappa_{\epsilon_0}/\chi_{\epsilon_0}^2)^2$ vs $(\chi_{\epsilon_0}/\kappa_{\epsilon_0}^{2/3})$. The pressure, heat transfer, and skin friction are markedly reduced by leading-edge bluntness. Thus, in the design of compression surfaces, a compromise must be reached between protecting the leading edge and recovering as much pressure as possible on the ramp. In flows strongly influenced by leading-edge bluntness, the skin-friction measurements were found to be the only accurate method of determining incipient separation. The wedge angle required to cause incipient separation is reduced by leading-edge bluntness and the measurements correlated well in the form $M\theta_{\rm incip}(\kappa_{\epsilon_L}/\chi_{\epsilon_L}^2) = 4.34(\chi_{\epsilon}/\kappa_{\epsilon}^{2/3})^{-7/5}$.

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